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The imaginary part of the static gluon propagator in an anisotropic (viscous) QCD plasma

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Abstract

We determine viscosity corrections to the retarded, advanced and symmetric gluon self energies and to the static propagator in the weak-coupling "hard loop" approximation to high-temperature QCD. We apply these results to calculate the imaginary part of the heavy-quark potential which is found to be smaller (in magnitude) than at vanishing viscosity. This implies a smaller decay width of quarkonium bound states in an anisotropic plasma.

Introduction: The retarded, advanced and symmetric gluon self energy in an equilibrated, weakly-coupled high-temperature plasma can be calculated within the “hard thermal loop” effective theory [1, 2]. However, the standard expressions receive corrections if the plasma undergoes (anisotropic) expansion and if its shear viscosity is non-zero. In this letter, we provide explicit expressions for the leading viscous corrections to the retarded, advanced and symmetric gauge-boson self energies and to the corresponding (static) resummed propagators. As an application of particular interest, we finally determine the corrections to the imaginary part of the heavy-quark (static) potential in a QCD plasma. This imaginary part provides a (contribution to the) width Γ of quarkonium bound states [3, 4, 5] which in turn determines their dissociation temperature: dissociation is expected to occur when the binding energy decreases, with increasing temperature, to $\sim\Gamma$ [6, 7]. It is therefore interesting to determine the effects of non-zero shear viscosity on the imaginary part of the potential.

Corrections to the thermal distributions in an anisotropic plasma: We consider a hot QCD plasma which, due to expansion and non-zero viscosity, exhibits a local anisotropy in momentum space. The phase-space distribution of particles is given by

$$f(\mathbf{p}) \;=\; f_{\rm iso}\left(\sqrt{\mathbf{p}^2+\xi(\mathbf{p}\cdot\mathbf{n})^2}\right) \tag{1}$$

$$\approx\; f_{\rm iso}(p)\left[1-\xi\frac{(\mathbf{p}\cdot\mathbf{n})^2}{2pT}\left(1\pm f_{\rm iso}(p)\right)\right]\;. \tag{2}$$

Thus, $f(\mathbf{p})$ is obtained from an isotropic distribution $f_{\rm iso}(|\mathbf{p}|)$ by removing particles with a large momentum component along $\mathbf{n}$, the direction of anisotropy [8]. We shall restrict ourselves here to a plasma close to equilibrium and so $f_{\rm iso}(p)$ is a thermal ideal-gas distribution equal to either a Bose distribution $n_B(p)$ or to a Fermi distribution $n_F(p)$, respectively. Equation (2) follows from an expansion in the anisotropy parameter ξ. The correction δf to the equilibrium distribution exhibits precisely the structure expected from viscous hydrodynamics for a fluid element expanding one-dimensionally along the direction $\mathbf{n}$, provided we identify [9]

$$\xi=\frac{10}{T\tau}\frac{\eta}{s}\;. \tag{3}$$

Note that this expression only holds true in the Navier-Stokes limit. In the general case, one can relate ξ to the shear tensor [10]. Here, $1/\tau$ denotes the expansion rate of the fluid element and η/s is the ratio of shear viscosity to entropy density. As usual in viscous hydrodynamics, the temperature T as well as the entropy density s appearing in Eqs. (2) and (3) are defined at equilibrium; all viscous corrections are accounted for explicitly by δf in Eq. (2).

Propagators in the Keldysh real-time formalism: We shall calculate the finite-temperature self energies and propagators using the well-known Keldysh real time formalism [11]. The propagators are then 2×2 matrices such as

$$D(P)=\begin{pmatrix}\frac{1}{P^2-m^2+i\epsilon} & 0\\ 0 & \frac{-1}{P^2-m^2-i\epsilon}\end{pmatrix}-2\pi i\,\delta(P^2-m^2)\begin{pmatrix} f_B & \Theta(-p_0)+f_B\\ \Theta(p_0)+f_B & f_B\end{pmatrix} \tag{4}$$

for a scalar field and

$$S(P)=(\not{P}+m)\quad\left[\begin{pmatrix}\frac{1}{P^2-m^2+i\epsilon} & 0\\ 0 & \frac{-1}{P^2-m^2-i\epsilon}\end{pmatrix}\right.$$

$$+2\pi i\,\delta(P^2-m^2)\,\left(\begin{array}{cc} f_F & -\Theta(-p_0)+f_F \\ -\Theta(p_0)+f_F & f_F \end{array}\right)\Bigg], \tag{5}$$

for a Dirac field. We use the following notation: $P=(p_0,\mathbf{p})$, $p=|\mathbf{p}|$. In equilibrium, the distribution functions f_B and f_F correspond to Bose or Fermi distribution functions, respectively. Away from equilibrium they need to be replaced by the corresponding non-equilibrium distributions from viscous hydrodynamics. It should be noted that Eqs. (4) and (5) are "bare" propagators, the hard loop resummation has yet to be performed.

The retarded, advanced and symmetric propagators can be obtained from the Keldysh representation (which satisfies $D_{11}-D_{12}-D_{21}+D_{22}=0$) via

$$D_R=D_{11}-D_{12}\ ,\ D_A=D_{11}-D_{21}\ ,\ D_F=D_{11}+D_{22}\ . \tag{6}$$

In momentum space, the explicit expressions are

$$\begin{aligned} D_R(P) &= \frac{1}{P^2-m^2+i\,\mathrm{sgn}(p_0)\epsilon}, \\ D_A(P) &= \frac{1}{P^2-m^2-i\,\mathrm{sgn}(p_0)\epsilon}, \\ D_F(P) &= -2\pi i\,(1+2f_B)\,\delta(P^2-m^2) \end{aligned} \tag{7}$$

for scalar bosons and

$$\begin{aligned} S_R(P) &= \frac{\not{P}+m}{P^2-m^2+i\,\mathrm{sgn}(p_0)\epsilon}, \\ S_A(P) &= \frac{\not{P}+m}{P^2-m^2-i\,\mathrm{sgn}(p_0)\epsilon}, \\ S_F(P) &= -2\pi i\,(\not{P}+m)\,(1-2f_F)\,\delta(K^2-m^2) \end{aligned} \tag{8}$$

for fermions, respectively.

In the real time formalism, similar relations hold for the self energies:

$$\Pi_{11}+\Pi_{12}+\Pi_{21}+\Pi_{22}=0 \tag{9}$$

and

$$\Pi_R=\Pi_{11}+\Pi_{12}\ ,\ \Pi_A=\Pi_{11}+\Pi_{21}\ ,\ \Pi_F=\Pi_{11}+\Pi_{22}\ . \tag{10}$$

Resummed photon (gluon) propagator: The resummed photon propagator can be determined from the Dyson-Schwinger equation

$$iD^*=iD+iD\,(-i\Pi)\,iD^*\,, \tag{11}$$

where the propagators and self energy are 2×2 matrices. D^* indicates a resummed propagator and D is the bare propagator.

Using the identities (6) for the bare and resummed propagators and (10) for the self energies, it is easy to show that

$$D^*{}_R=D_R+D_R\Pi_R D^*{}_R. \tag{12}$$

A similar expression holds for the advanced propagator. The resummed symmetric propagator satisfies

$$D^*{}_F = D_F + D_R\Pi_R D^*{}_F + D_F\Pi_A D^*{}_A + D_R\Pi_F D^*{}_A \ . \tag{13}$$

Using $D_F(P) = (1+2f_B)\,\mathrm{sgn}(p_0)\,[D_R(P) - D_A(P)]$ (this equation is true even in the non-equilibrium case [11]), the solution for $D^*{}_F$ can be expressed in the form

$$\begin{aligned} D^*{}_F(P) = \quad & (1+2f_B)\,\mathrm{sgn}(p_0)\,[D^*{}_R(P) - D^*{}_A(P)] \\ & + D^*{}_R(P)\,\{\Pi_F(P) - (1+2f_B)\,\mathrm{sgn}(p_0)\,[\Pi_R(P) - \Pi_A(P)]\}\,D^*{}_A(P) \ . \end{aligned} \tag{14}$$

In equilibrium we have $\Pi_F(P) = [1+2n_B(p_0)]\,\mathrm{sgn}(p_0)\,[\Pi_R(P) - \Pi_A(P)]$ and as a result the second term in Eq. (14) vanishes. However, this is no longer true out of equilibrium.

For the static potential we only require the temporal component of the gluon propagator. At leading order in ξ (or in the shear viscosity) the calculation of D^{*00} is easiest in Coulomb gauge. The temporal component decouples from the other components which simplifies the calculation significantly. In this gauge (with the gauge parameter $\eta = 0$) the bare or resummed propagators satisfy

$$K^i \cdot D^{0\,i} = 0 \ \ , i = 1, 2, 3 \tag{15}$$

as a consequence of $\partial_i A^i = 0$. In the isotropic case this reduces to $D^{0\,i} = 0$. We can then write Eq. (12) as

$$D^*{}^L_{R(0)} = D^L_R + D^L_R \Pi^L_{R(0)} D^*{}^L_{R(0)} \ . \tag{16}$$

Here, L denotes the temporal component, $D^L \equiv D^{00}$. A similar relation holds for the advanced propagator. Eq. (16) no longer holds for the anisotropic system due to breaking of isotropy in momentum space. However, for small anisotropy, we can expand the resummed propagators and self energies in ξ:

$$D^* = D^*_{(0)} + \xi D^*_{(1)} + O(\xi^2), \qquad \Pi = \Pi_{(0)} + \xi\Pi_{(1)} + O(\xi^2) \ . \tag{17}$$

The propagators to order ξ^0, $D^*_{(0)}$ (either retarded, advanced or symmetric) satisfy the equilibrium relations mentioned above. For the linear term of order ξ,

$$D^*{}_{R(1)} = D_R\,\Pi_{R(1)} D^*{}_{R(0)} + D_R\,\Pi_{R(0)} D^*{}_{R(1)} \ . \tag{18}$$

For the temporal component, we have

$$D^*{}^L_{R(1)} = (D_R)^{0\,\mu}\,(\Pi_{R(1)})_{\mu\,\nu}(D^*{}_{R(0)})^{\nu\,0} + (D_R)^{0\,\mu}\,(\Pi_{R(0)})_{\mu\,\nu}(D^*{}_{R(1)})^{\nu\,0} \ . \tag{19}$$

Since $(D_R)^{0\,i} = 0$, $(D^*{}_{R(0)})^{0\,i} = 0$ and $(\Pi_{R(0)})^{0\,i} \sim P^i$, according to Eq. (15), it follows that[1]

$$D^*{}^L_{R(1)} = D^L_R\,\Pi^L_{R(1)} D^*{}^L_{R(0)} + D^L_R\,\Pi^L_{R(0)} D^*{}^L_{R(1)} \ . \tag{20}$$

[1] The temporal component of the retarded propagator to order ξ^2, $D^*{}^L_{R(2)}$, fails to satisfy such a relation. In fact, it includes a product of $(D_R)^{0\,\mu}\,(\Pi_{R(1)})_{\mu\,\nu}(D^*{}_{R(1)})^{\nu\,0}$, but $(\Pi_{R(1)})^{0\,i}$ is not proportional to P^i. As a result, $\sum \Pi^{0\,i}_{R(1)} D^*{}^{i\,0}_{R(1)}$ doesn't give zero automatically. This term will therefore depend on the spatial components of the self-energy and propagator which makes the calculation more complicated.

Again, a similar relation holds for the advanced propagator. For the symmetric propagator, finally,

$$\begin{aligned} D^{*L}_{F(1)}(P) &= (1+2f_{B(0)})\,\mathrm{sgn}(p_0)\,[D^{*L}_{R(1)}(P)-D^{*L}_{A(1)}(P)] \\ &+\ 2f_{B(1)}\,\mathrm{sgn}(p_0)\,[D^{*L}_{R(0)}(P)-D^{*L}_{A(0)}(P)] \\ &+\ D^{*L}_{R(0)}(P)\,\{\Pi^L_{F(1)}(P)-(1+2f_{B(0)})\,\mathrm{sgn}(p_0)\,[\Pi^L_{R(1)}(P)-\Pi^L_{A(1)}(P)] \\ &-\ 2f_{B(1)}\,\mathrm{sgn}(p_0)\,[\Pi^L_{R(0)}(P)-\Pi^L_{A(0)}(P)]\}\,D^{*L}_{A(0)}(P)\ . \end{aligned} \tag{21}$$

We now proceed to calculate explicitly the photon/gluon self energies from which we obtain the propagators via the relations above. We employ the diagrammatic Hard Loop approach but we have checked that similar expression can be derived from Vlasov transport theory [12]. The contribution from the quark loop to the gluon self energy is of the form

$$\Pi^{\mu\nu}(P)=-\frac{i}{2}N_f g^2\int\frac{d^4K}{(2\pi)^4}tr\left[\gamma^\mu S(Q)\gamma^\nu S(K)\right], \tag{22}$$

where S denotes the “bare” quark propagator and $Q=K-P$. Summing the 11 and 12 components of the Keldysh representation leads to

$$\begin{aligned}\Pi^L_R(P)=-iN_f g^2\int\frac{d^4K}{(2\pi)^4}(q_0k_0+\mathbf{q}\cdot\mathbf{k})\quad&\Big[\tilde\Delta_F(Q)\tilde\Delta_R(K)+\tilde\Delta_A(Q)\tilde\Delta_F(K)\\ &+\tilde\Delta_A(Q)\tilde\Delta_A(K)+\tilde\Delta_R(Q)\tilde\Delta_R(K)\Big],\end{aligned} \tag{23}$$

where we neglect the fermion mass and write the fermion propagator as $S_{R,A,F}(K)\equiv \not{K}\tilde\Delta_{R,A,F}(K)$. The distribution function which appears in the symmetric propagator has the form

$$f_F(\mathbf{k})=n_F(k)-\xi n_F^2(k)\frac{e^{k/T}}{2kT}(\mathbf{k}\cdot\mathbf{n})^2+O(\xi^2)\ , \tag{24}$$

with $n_F(p)$ a Fermi-Dirac function. The last two terms of the integrand vanish after integration over k_0. Temperature independent terms will be dropped in the following. Shifting variables $K\to -K+P$ in the first term, we found that the first two terms give the same contributions to the final result. This is still true for a non-equilibrium distribution which satisfies $f(\mathbf{k})=f(-\mathbf{k})$. Then,

$$\begin{aligned}\Pi^L_R(P)=4\pi N_f g^2\quad&\int\frac{kdkd\Omega}{(2\pi)^4}f_F(\mathbf{k})\Big[(2k^2-p_0k-\mathbf{k}\cdot\mathbf{p})\frac{1}{P^2-2kp_0+2\mathbf{k}\cdot\mathbf{p}-i\,\mathrm{sgn}(k-p_0)\epsilon}\\ &+(2k^2+p_0k-\mathbf{k}\cdot\mathbf{p})\frac{1}{P^2+2kp_0+2\mathbf{k}\cdot\mathbf{p}-i\,\mathrm{sgn}(-k-p_0)\epsilon}\Big].\end{aligned} \tag{25}$$

Adopting the hard loop approximation, we assume that the internal momenta are of order T and therefore much larger than the external momentum which is of order gT [1]. The integrand in the square bracket can then be expanded in powers of the coupling and the leading term is of the form

$$\frac{2k^2}{-2kp_0+2\mathbf{k}\cdot\mathbf{p}-i\,\epsilon}+\frac{2k^2}{2kp_0+2\mathbf{k}\cdot\mathbf{p}+i\,\epsilon}\ . \tag{26}$$

It can be easily shown that after integrating over $d\Omega$, this contribution vanishes. The next to leading term comes from the following 4 terms in the expansion of the integrand in the square bracket

$$\frac{-p_0 k - \mathbf{k}\cdot\mathbf{p}}{-2kp_0 + 2\mathbf{k}\cdot\mathbf{p} - i\,\epsilon} + \frac{p_0 k - \mathbf{k}\cdot\mathbf{p}}{2kp_0 + 2\mathbf{k}\cdot\mathbf{p} + i\,\epsilon}$$
$$-\frac{2k^2P^2}{(-2kp_0 + 2\mathbf{k}\cdot\mathbf{p} - i\,\epsilon)^2} - \frac{2k^2P^2}{(2kp_0 + 2\mathbf{k}\cdot\mathbf{p} + i\,\epsilon)^2}. \tag{27}$$

After integrating over $d\Omega$, we find that the first two terms in Eq. (27) give the same result; the last two terms also contribute equally. As a result, the retarded self energy can be expressed as

$$\Pi_R^L(P) = \frac{4\pi N_f g^2}{(2\pi)^4}\int k dk d\Omega f_F(\mathbf{k}) \frac{1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2}{(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}} + \frac{p_0+i\,\epsilon}{p})^2}. \tag{28}$$

Here, three-momenta with a hat denote unit vectors.

Using the distribution function (24) it is now straightforward to calculate the retarded self energy:

$$\Pi_R^L(P) = \frac{g^2}{2\pi^2} N_f \sum_{i=0,1} \int_0^\infty k\,\Phi_{(i)}(k) dk \int_{-1}^1 \Psi_{(i)}(s) ds\ , \tag{29}$$

with

$$\begin{aligned}
\Phi_{(0)}(k) &= n_F(k)\,, \\
\Phi_{(1)}(k) &= -\xi n_F^2(k)\,\frac{e^{k/T}k}{2T}\,, \\
\Psi_{(0)}(s) &= \frac{1-s^2}{(s+\frac{p_0+i\epsilon}{p})^2}\,, \\
\Psi_{(1)}(s) &= \cos^2\alpha\,\frac{s^2(1-s^2)}{(s+\frac{p_0+i\epsilon}{p})^2} + \frac{\sin^2\alpha}{2}\,\frac{(1-s^2)^2}{(s+\frac{p_0+i\epsilon}{p})^2}\ .
\end{aligned} \tag{30}$$

Here, α is the angle between $\mathbf{n}$ and $\mathbf{p}$ and $s \equiv \hat{\mathbf{k}}\cdot\hat{\mathbf{p}}$. This integral can be performed analytically and the leading contribution for the isotropic term is

$$\Pi_{R(0)}^L(P) = \frac{g^2}{2\pi^2} N_f \int_0^\infty k\,\Phi_{(0)}(k) dk \int_{-1}^1 \Psi_{(0)}(s) ds = N_f \frac{g^2T^2}{6}\left(\frac{p_0}{2p}\ln\frac{p_0+p+i\epsilon}{p_0-p+i\epsilon} - 1\right)\ . \tag{31}$$

To order $\sim \xi$, the result is

$$\begin{aligned}
\Pi_{R(1)}^L(P) &= \frac{g^2}{2\pi^2} N_f \int_0^\infty k\,\Phi_{(1)}(k) dk \int_{-1}^1 \Psi_{(1)}(s) ds \\
&= N_f \frac{g^2T^2}{6}\left(\frac{1}{6} + \frac{\cos(2\alpha)}{2}\right) + \Pi_{R(0)}^L(P)\left[\cos(2\alpha) - \frac{p_0^2}{2p^2}(1+3\cos(2\alpha))\right]\ .
\end{aligned} \tag{32}$$

In the Hard Loop limit the gluon-loop contributions to the gluon self energy have the same structure as the one due to a quark loop. (For the same reason, the gluon self energy in

the Hard Loop approximation is gauge invariant.) We can simply replace $N_f \frac{g^2 T^2}{6}$ by m_D^2 to generalize the QED result to QCD. In the following, the Debye mass m_D^2 is to be understood as

$$m_D^2 = -\frac{g^2}{2\pi^2} \int_0^\infty dk\, k^2 \, \frac{df_{\rm iso}(k)}{dk} \ . \tag{33}$$

This is independent of ξ as all viscous corrections shall be written explicitly. The isotropic distribution function $f_{\rm iso}$ in our case is just a sum of the Fermi and Bose distributions (with appropriate prefactors counting the number of degrees of freedom [8]). For N_f massless quark flavors and N_c colors,

$$m_D^2 = \frac{g^2 T^2}{6}(N_f + 2N_c) \ . \tag{34}$$

Analogously, the advanced self energy is given by

$$\Pi^L_{A(0)}(P) = m_D^2 \left(\frac{p_0}{2p} \ln \frac{p_0 + p - i\epsilon}{p_0 - p - i\epsilon} - 1 \right), \tag{35}$$

and

$$\Pi^L_{A(1)}(P) = m_D^2 \left(\frac{1}{6} + \frac{\cos(2\alpha)}{2} \right) + \Pi^L_{A(0)}(P) \left[\cos(2\alpha) - \frac{p_0^2}{2p^2} \left(1 + 3\cos(2\alpha)\right) \right] \ . \tag{36}$$

Using Eqs. (16) and (20), it is straightforward to obtain the temporal component of the retarded propagator in Coulomb gauge

$$D^{*L}_{R(0)} = \left(p^2 - m_D^2 \left(\frac{p_0}{2p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon} - 1 \right) \right)^{-1} , \tag{37}$$

$$D^{*L}_{R(1)} = \frac{m_D^2 (\frac{1}{6} + \frac{\cos(2\alpha)}{2}) + \Pi^L_{R(0)} [\cos(2\alpha) - \frac{p_0^2}{2p^2}(1 + 3\cos(2\alpha))]}{(p^2 - m_D^2 (\frac{p_0}{2p} \ln \frac{p_0+p+i\epsilon}{p_0-p+i\epsilon} - 1))^2} \ . \tag{38}$$

Similar results can be obtained for the advanced propagator. These results are identical to the ones obtained within the transport theory approach.

Next we calculate Π^L_F within the hard loop approximation. Eq. (14) shows that this quantity is necessary to obtain the resummed symmetric propagator out of equilibrium. Summing the 11 and 22 components of the Keldysh representation, we obtain

$$\begin{aligned} \Pi^L_F(P) = -iN_f g^2 \int \frac{d^4K}{(2\pi)^4} (q_0 k_0 + \mathbf{q} \cdot \mathbf{k}) \quad & \Big[\tilde{\Delta}_F(Q) \tilde{\Delta}_F(K) - (\tilde{\Delta}_R(Q) - \tilde{\Delta}_A(Q)) \\ & \times \ (\tilde{\Delta}_R(K) - \tilde{\Delta}_A(K)) \Big]. \end{aligned} \tag{39}$$

Using $\tilde{\Delta}_R(Q) - \tilde{\Delta}_A(Q) = -2\pi i \, \mathrm{sgn}(q_0) \delta(Q^2)$ and the hard loop approximation, we find that the symmetric self energy can be expressed as

$$\Pi^L_F(P) = 4iN_f g^2 \pi^2 \int \frac{k^2 dk d\Omega}{(2\pi)^4} f_F(\mathbf{k})(f_F(\mathbf{k}) - 1) \frac{2}{p} \Big[\delta(s + \frac{p_0}{p}) + \delta(s - \frac{p_0}{p}) \Big]. \tag{40}$$

Again, we can expand the anisotropic distribution function to order ξ and finally arrive at

$$
\begin{aligned}
\Pi^L_{F(0)}(P) &= -2\pi i\, m_D^2 \frac{T}{p}\Theta(p^2-p_0^2)\,,\\
\Pi^L_{F(1)}(P) &= \frac{3}{2}\pi i\, m_D^2 \frac{T}{p}\left(\sin^2\alpha + (3\cos^2\alpha - 1)\frac{p_0^2}{p^2}\right)\Theta(p^2-p_0^2)\ .
\end{aligned}
\tag{41}
$$

Next, we can calculate the symmetric propagator. We first consider the isotropic case with $\xi = 0$ and perform a Taylor expansion assuming $p_0 \to 0$:

$$
D^{*L}_{R(0)}(P) - D^{*L}_{A(0)}(P) = \frac{m_D^2}{2p}\frac{-2\pi i}{(p^2+m_D^2)^2}\,p_0\ , \tag{42}
$$

which follows from

$$
\lim_{p_0\to 0}(\ln\frac{p_0+p+i\,\epsilon}{p_0-p+i\,\epsilon} - \ln\frac{p_0+p-i\,\epsilon}{p_0-p-i\,\epsilon}) = -2\pi i\ . \tag{43}
$$

Similarly, when p_0 is small, the distribution function of on-shell thermal gluons is

$$
(1+2n_B)\,\mathrm{sgn}(p_0) = \frac{2T}{p_0}\ . \tag{44}
$$

In the above equation, terms that do not contribute to the symmetric propagator in the static limit have been neglected. Finally, we can determine the temporal component of the symmetric propagator explicitly in the isotropic limit:

$$
D^{*L}_{F(0)}(p_0=0) = -\frac{2\pi i T m_D^2}{p\,(p^2+m_D^2)^2}\ . \tag{45}
$$

We now consider the contribution to order ξ. From Eq. (21), the gluon distribution function can be expanded as

$$
f_B = f_{B(0)} + \xi f_{B(1)} = \frac{T}{|p_0|} - \frac{T\cos^2\alpha}{2|p_0|}\xi. \tag{46}
$$

There are 4 contributions at linear order of ξ as shown in Eq. (21). The calculation is similar to the isotropic case. In addition, we need the following Taylor expansion to linear order in p_0:

$$
\begin{aligned}
D^{*L}_{R(1)}(P) - D^{*L}_{A(1)}(P) &= -2\pi i\left[\frac{m_D^4(1-3\cos(2\alpha))}{6p(p^2+m_D^2)^3} + \frac{m_D^2\cos(2\alpha)}{2p(p^2+m_D^2)^2}\right]p_0\ ,\\
\Pi^{*L}_{R(1)}(P) - \Pi^{*L}_{A(1)}(P) &= \frac{-\pi i m_D^2\cos(2\alpha)}{p}\,p_0\ .
\end{aligned}
\tag{47}
$$

Now the $\mathcal{O}(\xi)$ term of the symmetric propagator in the static limit can be derived from the above equations

$$
D^{*L}_{F(1)}(p_0=0) = \frac{3\pi i T m_D^2}{2p\,(p^2+m_D^2)^2}\sin^2\alpha - \frac{4\pi i T m_D^4}{p\,(p^2+m_D^2)^3}(\sin^2\alpha - \frac{1}{3})\ . \tag{48}
$$

Heavy quark potential in an anisotropic plasma: In the real time formalism, the static heavy quark potential due to one gluon exchange can be determined through the Fourier transform of the physical "11" component of the gluon propagator in the static limit:

$$\begin{aligned} V(\mathbf{r},\xi) &= -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3}\,(e^{i\mathbf{p}\cdot\mathbf{r}}-1)\left(D^{*L}(p_0=0,\mathbf{p},\xi)\right)_{11} \\ &= -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3}\,(e^{i\mathbf{p}\cdot\mathbf{r}}-1)\frac{1}{2}\left(D^{*L}_R + D^{*L}_A + D^{*L}_F\right) \\ &= -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3}\,(e^{i\mathbf{p}\cdot\mathbf{r}}-1)\frac{1}{2}\left(D^{*L}_R + D^{*L}_A\right) \\ &- \; g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3}\,(e^{i\mathbf{p}\cdot\mathbf{r}}-1)\frac{1}{2}D^{*L}_F \; . \end{aligned} \tag{49}$$

In the static limit, $\frac{1}{2}\left(D^{*L}_R + D^{*L}_A\right) = D^{*L}_R = D^{*L}_A$. The Fourier transform of this quantity gives the real part of the potential which has been discussed previously in Refs. [13, 14] and which determines the quarkonium binding energies. Here, we instead consider the imaginary part which comes from the Fourier transform of the symmetric propagator. From Eqs. (45) and (48), the isotropic contribution is given by

$$\mathbf{Im}\; V_{(0)}(r) = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3}(e^{i\mathbf{p}\cdot\mathbf{r}}-1)\frac{-\pi T m_D^2}{p\,(p^2+m_D^2)^2} = -\frac{g^2 C_F T}{4\pi}\,\phi(\hat{r}) \; , \tag{50}$$

with

$$\phi(\hat{r}) = 2\int_0^\infty dz \frac{z}{(z^2+1)^2}\left[1-\frac{\sin(z\,\hat{r})}{z\,\hat{r}}\right] \; , \tag{51}$$

and $\hat{r} \equiv r\,m_D$. This result has been derived before in Refs. [3, 4]. The term of order ξ can be expressed as

$$\begin{aligned} \mathbf{Im}\;\xi V_{(1)}(\mathbf{r}) &= -g^2 C_F\,\xi \int \frac{d^3\mathbf{p}}{(2\pi)^3}\,(e^{i\mathbf{p}\cdot\mathbf{r}}-1) \\ &\times \left[\frac{3\pi T m_D^2}{4p\,(p^2+m_D^2)^2}\sin^2\alpha - \frac{2\pi T m_D^4}{p\,(p^2+m_D^2)^3}(\sin^2\alpha - \frac{1}{3})\right] \\ &= \frac{g^2 C_F \xi T}{4\pi}\left[\psi_1(\hat{r},\theta)+\psi_2(\hat{r},\theta)\right] \; , \end{aligned} \tag{52}$$

where θ is the angle between $\mathbf{r}$ and $\mathbf{n}$ and

$$\psi_1(\hat{r},\theta) \;=\; \int_0^\infty dz \frac{z}{(z^2+1)^2}\left(1-\frac{3}{2}\left[\sin^2\theta\frac{\sin(z\,\hat{r})}{z\,\hat{r}} + (1-3\cos^2\theta)G(\hat{r},z)\right]\right) \; , \tag{53}$$

$$\psi_2(\hat{r},\theta) \;=\; -\int_0^\infty dz \frac{\frac{4}{3}z}{(z^2+1)^3}\left(1-3\left[\left(\frac{2}{3}-\cos^2\theta\right)\frac{\sin(z\,\hat{r})}{z\,\hat{r}} + (1-3\cos^2\theta)G(\hat{r},z)\right]\right) \tag{54}$$

with

$$G(\hat{r},z) = \frac{\hat{r}z\cos(\hat{r}z)-\sin(\hat{r}z)}{(\hat{r}z)^3} \; . \tag{55}$$

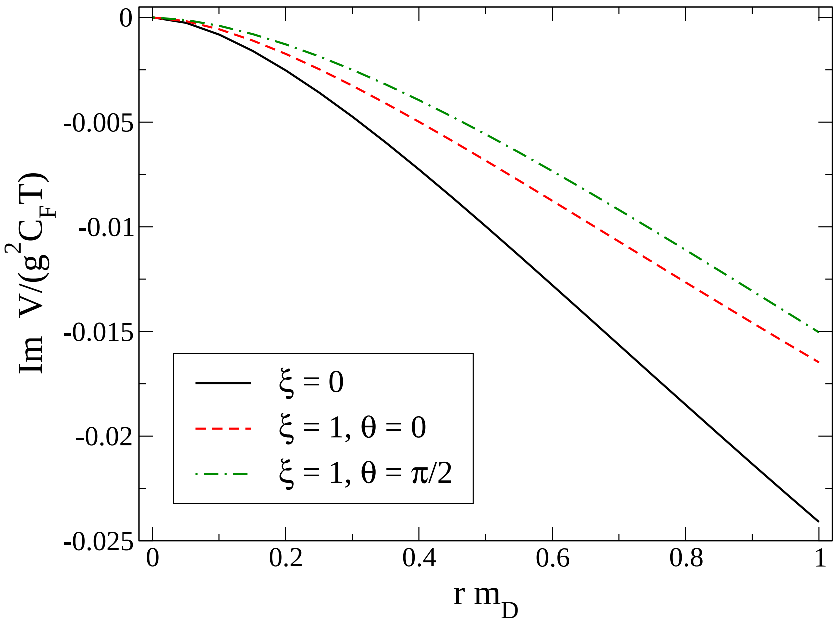

FIG. 1: Imaginary part of the static potential as a function of distance ($\hat{r} \equiv r\, m_D$). The vertical axis is Im $V/(g^2 C_F T)$. The curves, from bottom to top, correspond to an anisotropy of $\xi = 0$ and $\xi = 1$, $\theta = 0$, $\theta = \pi/2$.

The result is shown in fig. 1. The imaginary part decreases with ξ (or with the viscosity, respectively). When $\hat{r}$ is small, we can expand the potential. This is relevant for bound states of very heavy quarks whose Bohr radii $\sim 1/(g^2 M_Q)$ are smaller than the Debye length $1/m_D$. For the imaginary part, at leading order, the corresponding functions take the following forms:

$$\begin{aligned} \phi(\hat{r}) &= -\frac{1}{9}\,\hat{r}^2(-4+3\gamma_E+3\ln\hat{r})\ , \\ \psi_1(\hat{r},\theta) &= \frac{1}{600}\,\hat{r}^2[123-90\gamma_E-90\ln\hat{r}+\cos(2\theta)(-31+30\gamma_E+30\ln\hat{r})]\ , \\ \psi_2(\hat{r},\theta) &= \frac{1}{90}\,\hat{r}^2(-4+3\cos(2\theta))\ , \end{aligned} \tag{56}$$

where γ_E is the Euler-Gamma constant. At leading logarithmic order then

$$\mathbf{Im}\ V(\mathbf{r},\xi) = -\frac{g^2 C_F T}{4\pi}\,\hat{r}^2\,\ln\frac{1}{\hat{r}}\left(\frac{1}{3}-\xi\frac{3-\cos(2\theta)}{20}\right)\ . \tag{57}$$

Treating the imaginary part of the potential as a perturbation of the vacuum Coulomb potential provides an estimate for the decay width,

$$\begin{aligned} \Gamma &= \frac{g^2 C_F T}{4\pi}\int d^3\mathbf{r}\,\left|\Psi(r)\right|^2\,\hat{r}^2\ln\frac{1}{\hat{r}}\left(\frac{1}{3}-\xi\frac{3-\cos(2\theta)}{20}\right) \\ &= \frac{16\pi T}{g^2 C_F}\frac{m_D^2}{M_Q^2}\left(1-\frac{\xi}{2}\right)\ln\frac{g^2 C_F M_Q}{8\pi m_D}\ . \end{aligned} \tag{58}$$

Here, M_Q is the quark mass and $\Psi(r)$ is the ground state Coulomb wave function. Thus, at leading order in the deviation from equilibrium (i.e., viscosity) the quarkonium decay width

is smaller. For a moderate anisotropy $\xi \simeq 1$, Γ decreases by about 50% as compared to an ideal, fully equilibrated plasma.

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Note added: While this paper was in the final stages of preparation, another paper appeared where the imaginary part of the heavy-quark potential at non-zero plasma anisotropy is being considered [15]. The authors point out that the $\mathcal{O}(\xi)$ correction contributes already at leading non-trivial order to the quarkonium decay width, which agrees with our finding. Their result differs from ours (numerically) but can be reproduced if the second contribution on the r.h.s. of Eq. (14) is omitted. From private communication with M. Laine, this appears to be due to a different setup of the non-equilibrium system: Ref [15] considers a situation where the soft gluons are in equilibrium at a temperature T while the hard gluons are out of equilibrium and are characterized by a different hard scale T'. In contrast, we assume that the deviation from equilibrium follows viscous hydrodynamics, Eq. (2).

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